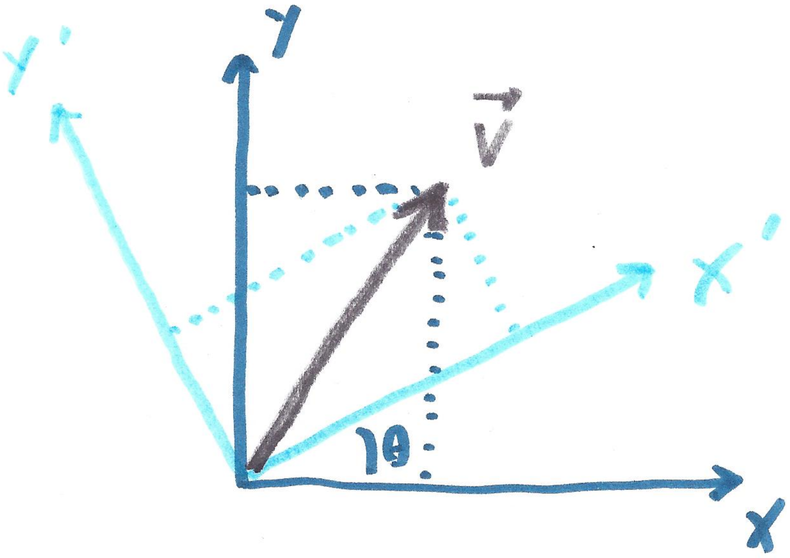
R.C. (1907 - 1913) (1)

GOAL: COVARIANZA UNIVERSALE
LEGGI NOTE 1907



$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

MATRICE TRASFORMAZIONE $\Lambda_{\alpha\beta}$

$$x'_\alpha = \sum_\beta \underbrace{\Lambda_{\alpha\beta}}_{\frac{\partial x'_\alpha}{\partial x_\beta}} x_\beta \qquad \begin{pmatrix} \frac{\partial x'_1}{\partial x_1} & \frac{\partial x'_2}{\partial x_1} \\ \frac{\partial x'_1}{\partial x_2} & \frac{\partial x'_2}{\partial x_2} \end{pmatrix}$$

QUANTITA' <u>TENSORIALI</u>

SCALARE $s' = s$

VETTORE $V'_\alpha = \sum_{\beta} \Lambda_{\alpha\beta} V_\beta$

MATRICE $W'_{\alpha\beta} = \sum_{\delta} \sum_{\gamma} \Lambda_{\alpha\gamma} \Lambda_{\beta\delta} W_{\gamma\delta}$

TENSORE A TANTI INDICI

$$T_{\alpha\beta\gamma\dots} = \sum_{\mu\nu\rho\dots} \Lambda_{\alpha\mu} \Lambda_{\beta\nu} \Lambda_{\gamma\rho} \dots T_{\mu\nu\rho\dots}$$

EQUAZIONE COVARIANTE

$$M_{\alpha\beta} = T_{\alpha\beta} \qquad V_\alpha = A_\beta$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

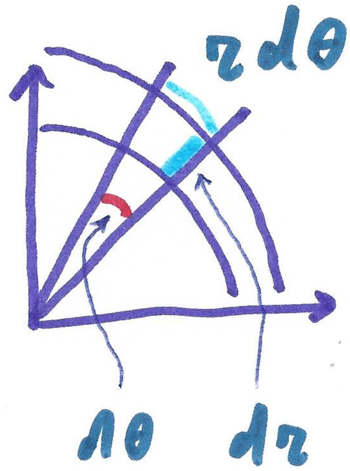
$$M'_{\alpha\beta} = T'_{\alpha\beta} \qquad V'_\alpha = V'_\beta$$

METRICA

GALILEO
$$\begin{cases} t' = t \\ |\vec{r}_2\,' - \vec{r}_1\,'| = |\vec{r}_2 - \vec{r}_1| \end{cases}$$
(eventi simultanei 1 e 2)

$$\left.\begin{aligned} dl^2 &= dx^2 + dy^2 = \\ &= (dx, dy) \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} \\ &= \sum_{\alpha\beta} g_{\alpha\beta}\, dx_\alpha\, dx_\beta \end{aligned}\right\} \text{CARTESIAN}$$

POLARI
$$\begin{cases} dl^2 = dr^2 + r^2 d\theta^2 \\ \bullet\ g_{\alpha\beta} = \begin{pmatrix} 1 & \\ & r^2 \end{pmatrix} \end{cases}$$



USANDO CORRETTE TRASF. $g_{\alpha\beta}$

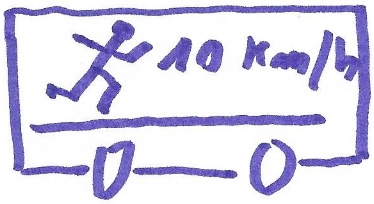
$$(dl^2)_{POL} = (dl^2)_{CART}$$

RELATIVITÀ SPECIALE (EINSTEIN LORENTZ MINKOWSKY 1905)

$t \neq t'$, $|\vec{r}_1 - \vec{r}_2| \neq |\vec{r}_1' - \vec{r}_2'|$

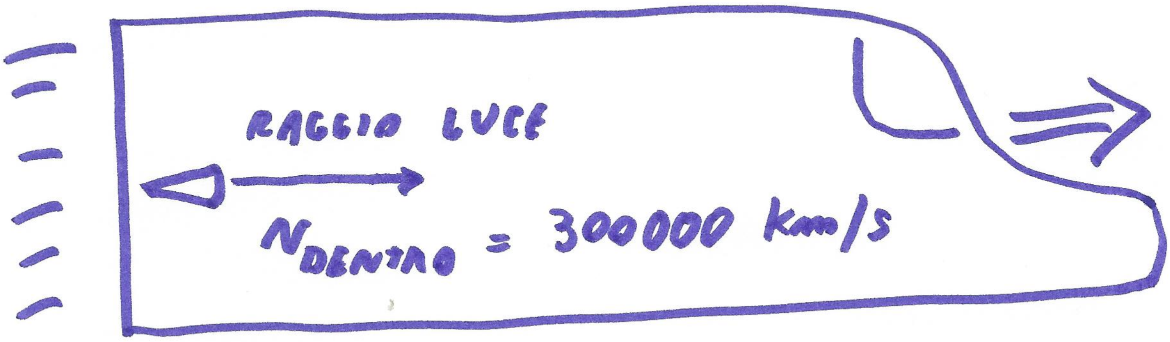
SISTEMI INERZIALI (MOTO RELATIVO NON ACCELERATO, COORD. CARTESIANE)

$$\boxed{V_{LUCE\ VUOTO} \equiv C \quad \text{SEMPRE}}$$



100 Km/h

$v_{UOMO\ VISTA\ DA\ FUORI} = 110$ Km/h

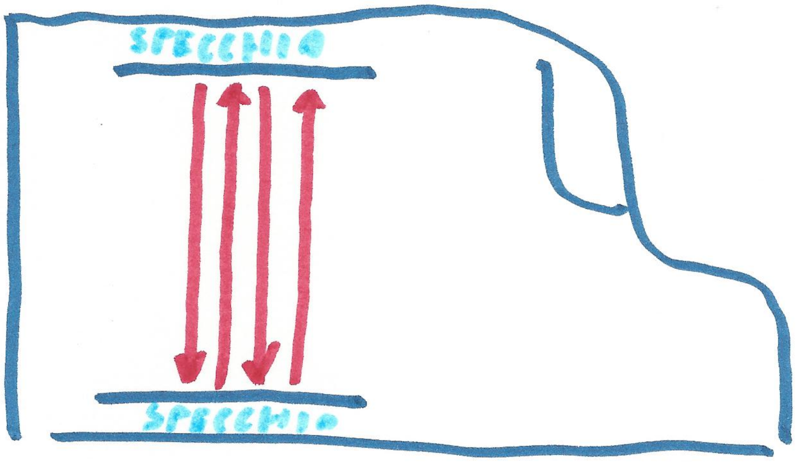


290000 Km/s

$v_{FUORI} = 300000$ Km/s $\ll$ 300000+290000 Km/s

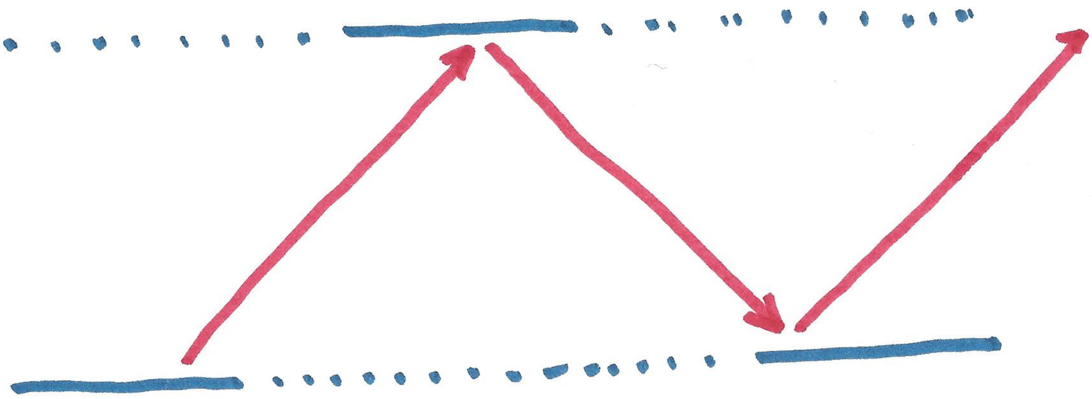
IMPOSSIBILE $\left(\frac{\Delta x}{\Delta t}\right)_{FUORI} = \left(\frac{\Delta x}{\Delta t}\right)_{DENTRO}$

DENTRO



RIFLESSIONI
MULTIPLE
LUCE

FUORI



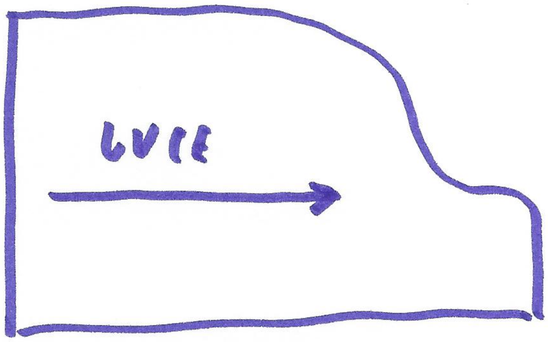
PERCORSO LUCE FUORI $>$ PERCORSO DENTR

$\Updownarrow$

TEMPO FUORI $>$ TEMPO DENTRO

COSÌ CHE $\left(\frac{\Delta l}{\Delta t}\right)_{FUORI} = \left(\frac{\Delta l}{\Delta t}\right)_{DENTRO}$

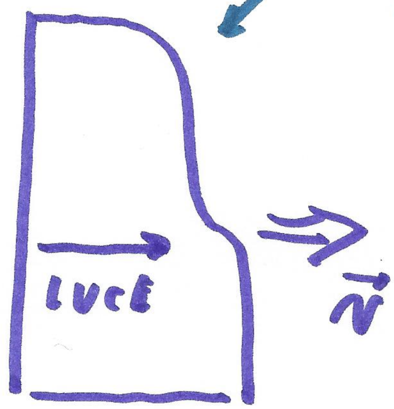
DENTRO



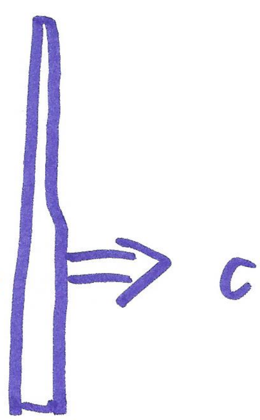
FUORI

CONTRAZIONE LONGITUDINALE

(DIMENSIONI TRASVERSE INALTERATE)



AL LIMITE $N \simeq C$

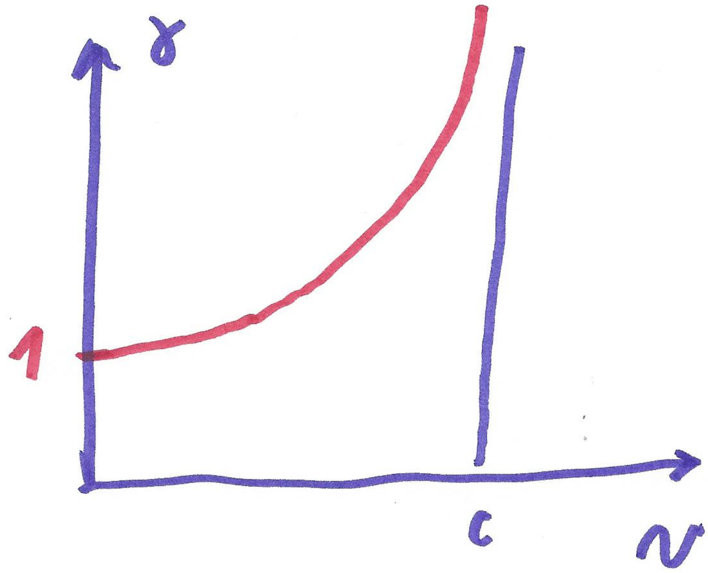


LA LUCE EMESSA DENTRO NON FA STRADA "AGGIUNTIVA"

FATTORE CONTRAZIONE γ

DILATAZIONE TEMPI $1/\gamma$

$$1/\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$



c = costante $\Longleftrightarrow$ $ds = ds'$

INVARIANTE UNIVERSALE SISTEMI INERZIALI

$$ds^2 = c^2 dt^2 - dl^2 =$$

$$(dct, dx, dy, dz) \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} dct \\ dx \\ dy \\ dz \end{pmatrix}$$

$g_{\alpha\beta}$

4-TENSORE METRICO

4 - VETTORI

$$(ct, x, y, z) \equiv (x^0, x^1, x^2, x^3) \equiv x^\mu$$

$$\left(\frac{E}{c}, p_x, p_y, p_z\right) \equiv (p^0, p^1, \dots) \equiv p^\mu$$

$$(\gamma c, \gamma v_x, \gamma v_y, \gamma v_z) \equiv v^\mu = \frac{dx^\mu}{d\tau}$$

$d\tau$ = TEMPO PROPRIO =

$= \frac{ds}{c}$ SE $dl = 0$

$$ds^2 = c^2 dt^2 - \cancel{dl^2}$$

SCRITTURE COVARIANTI

$$\frac{dv^\mu}{d\tau} = 0$$ PRINCIPIO INERZIA

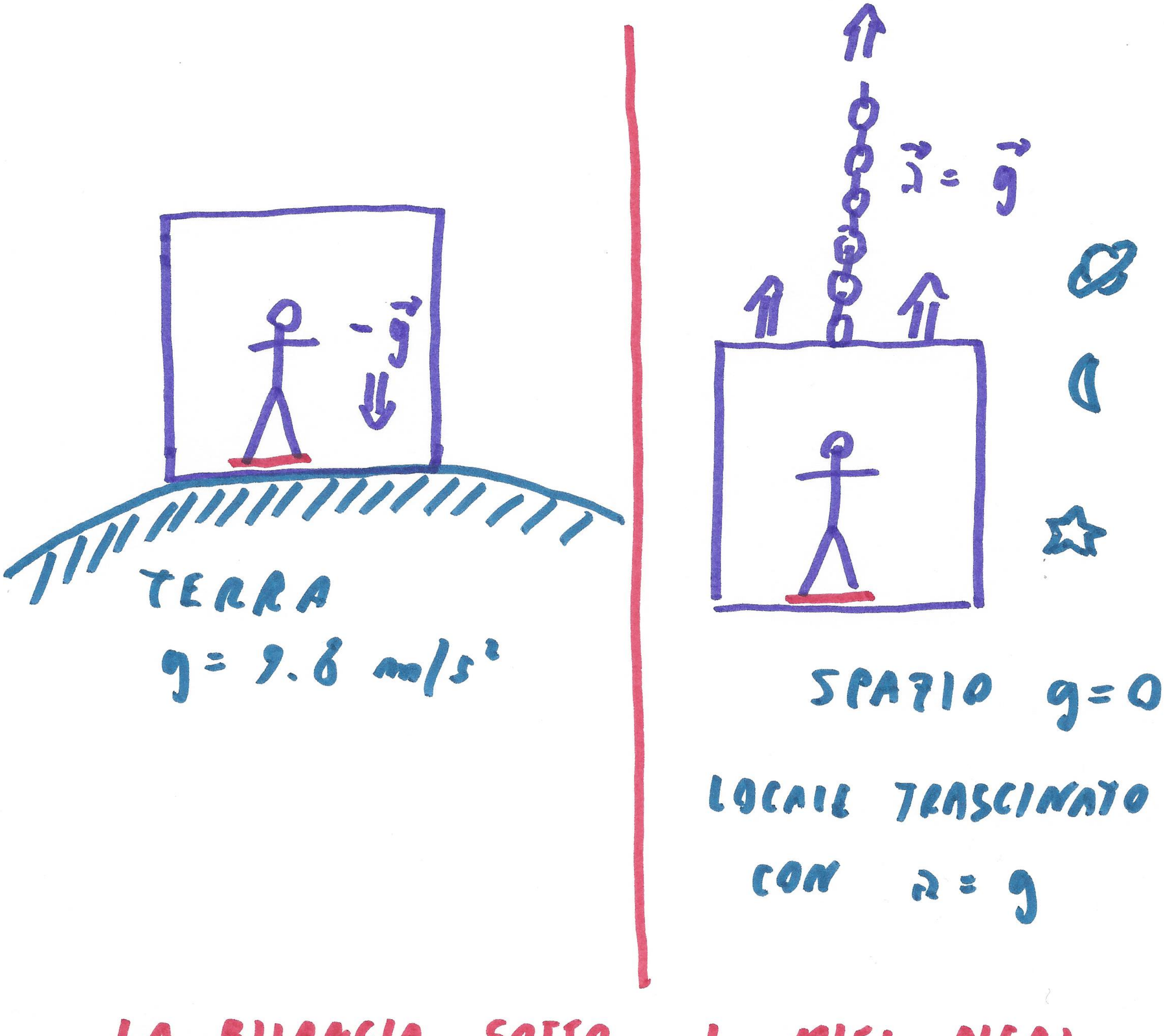
$$\frac{dp^\mu}{d\tau} = F^\mu$$ $F = ma$

OBIETTIVO EINSTEIN : GENERALIZZARE COVARIANZA

1907 — 1915

PRINCIPIO DI PARTENZA : EQUIVALENZA

(CORRETTO MA NON SUFFICIENTE)

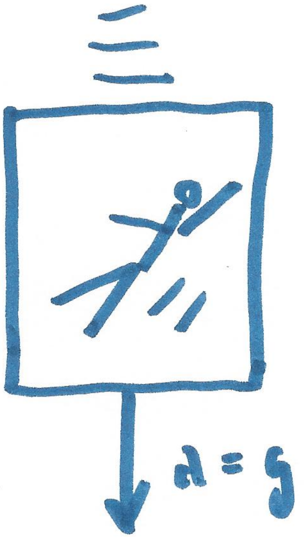


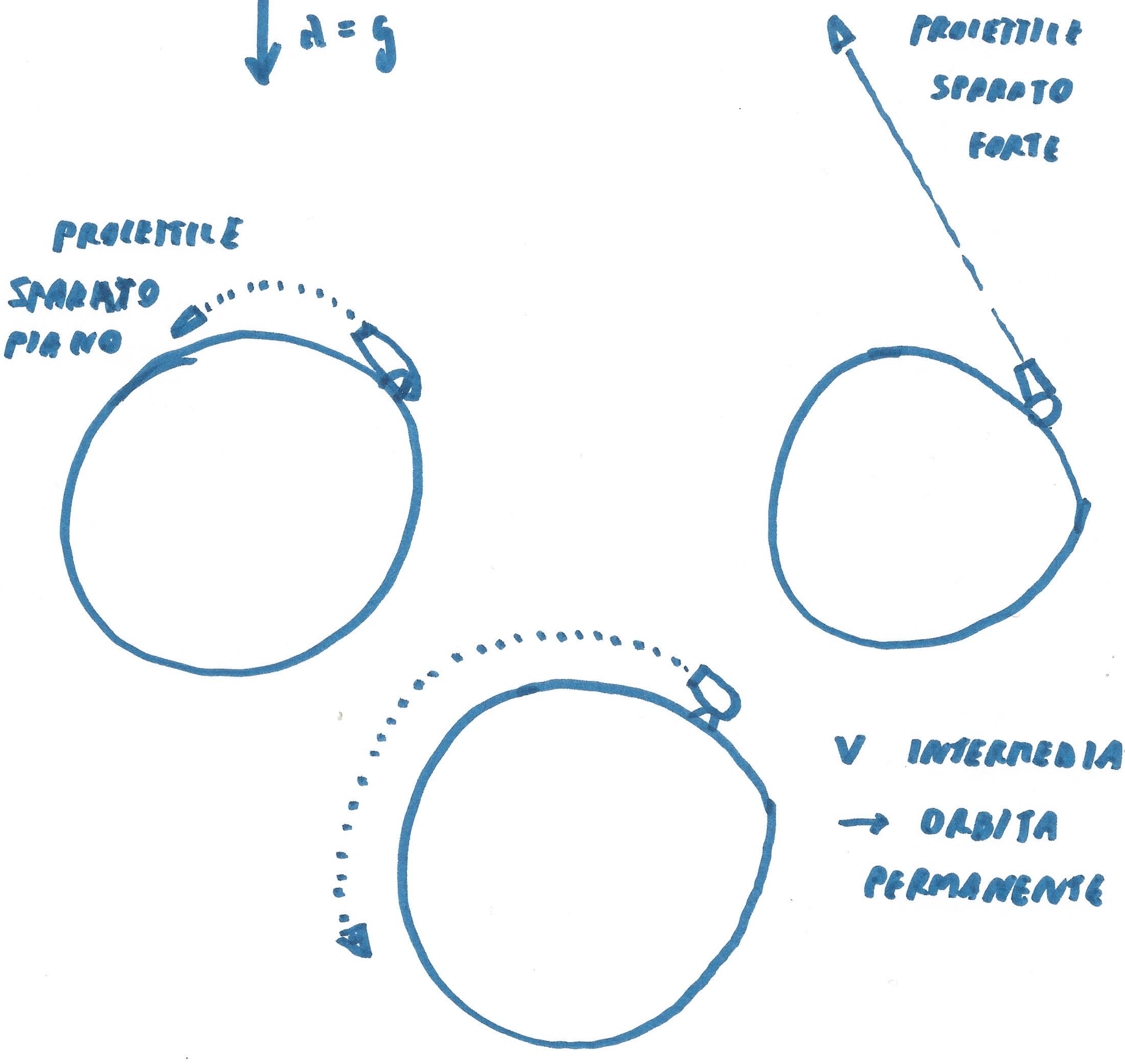
LA BILANCIA SOTTO I MIEI PIEDI

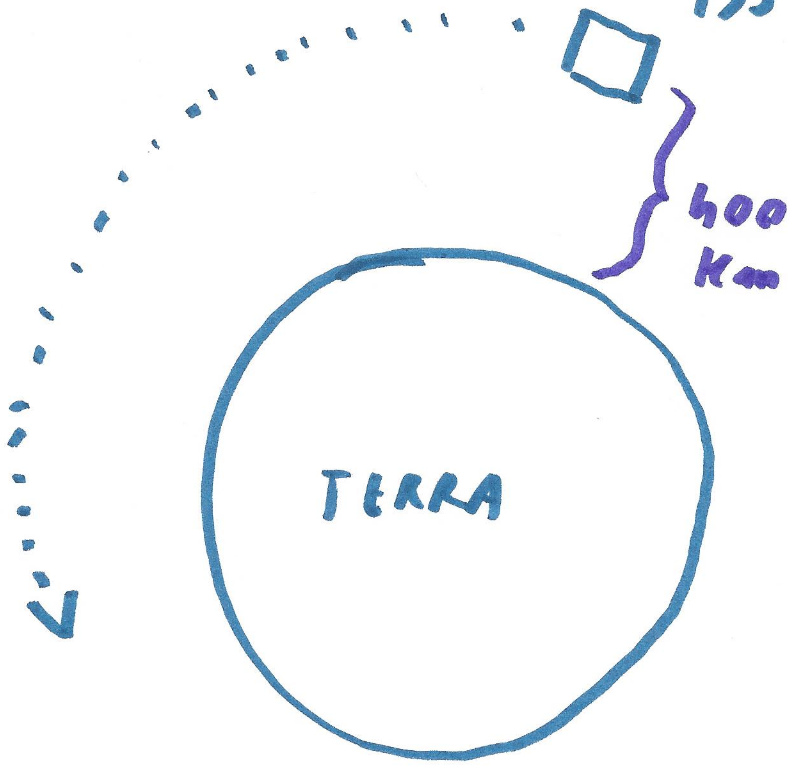
MISURA 67 Kg IN ENTRAMBI

ASCENSORI CADUTA LIBERA:

g sparisce

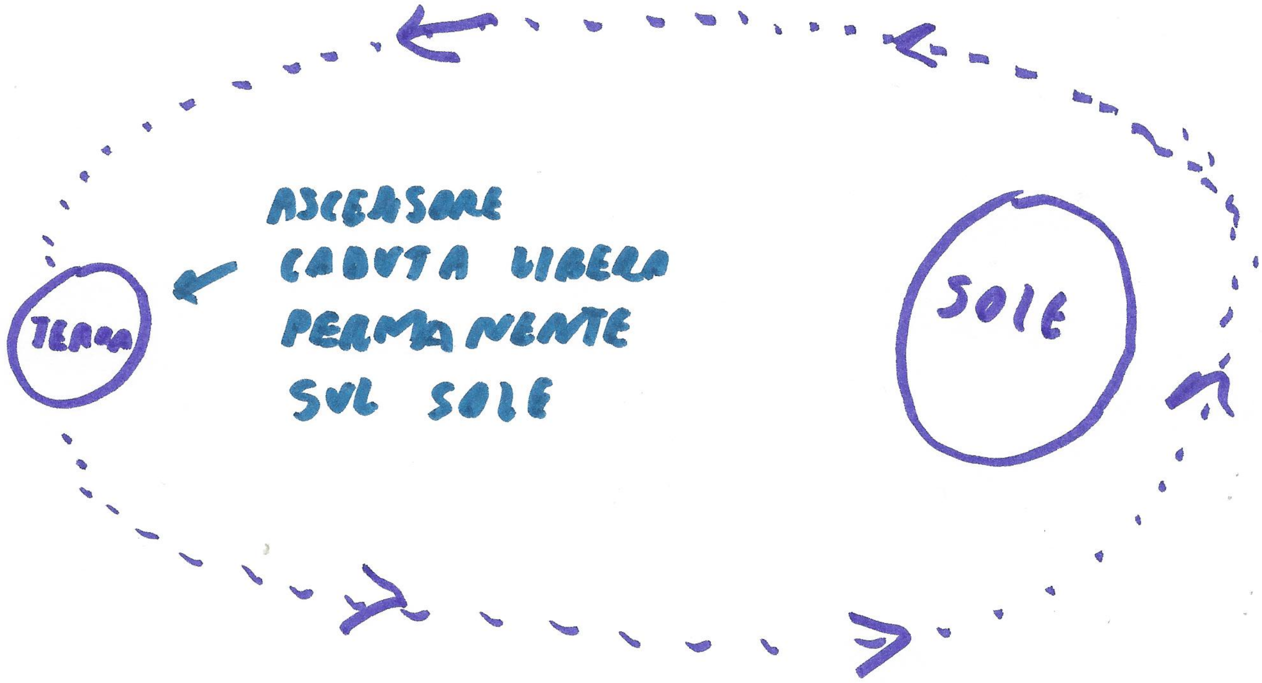




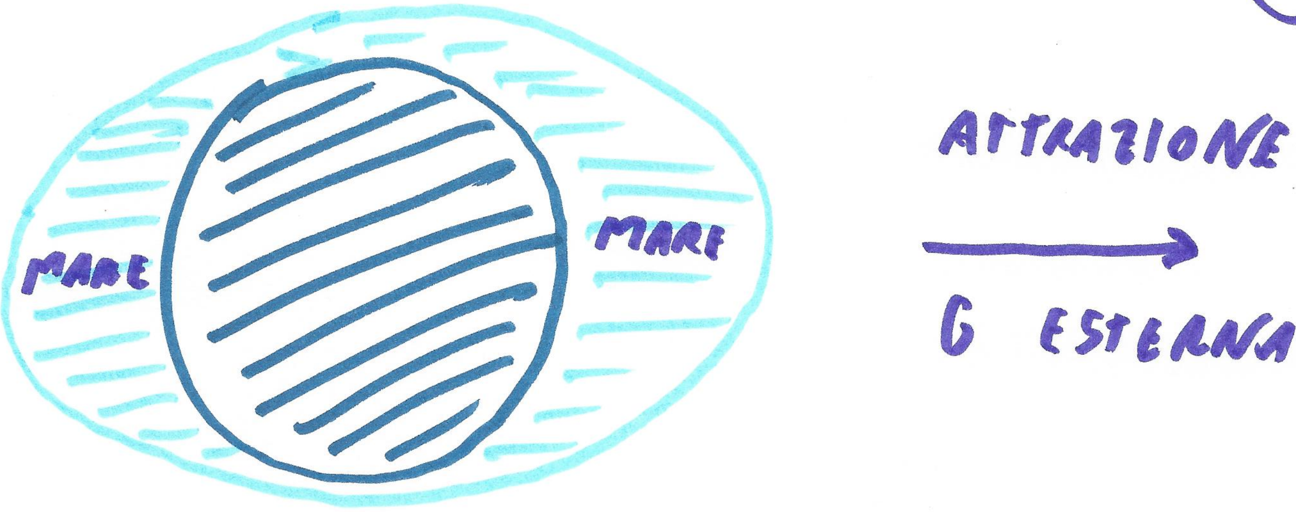


ISS = ASCENSORE
CADUTA LIBERA
PERMANENTE

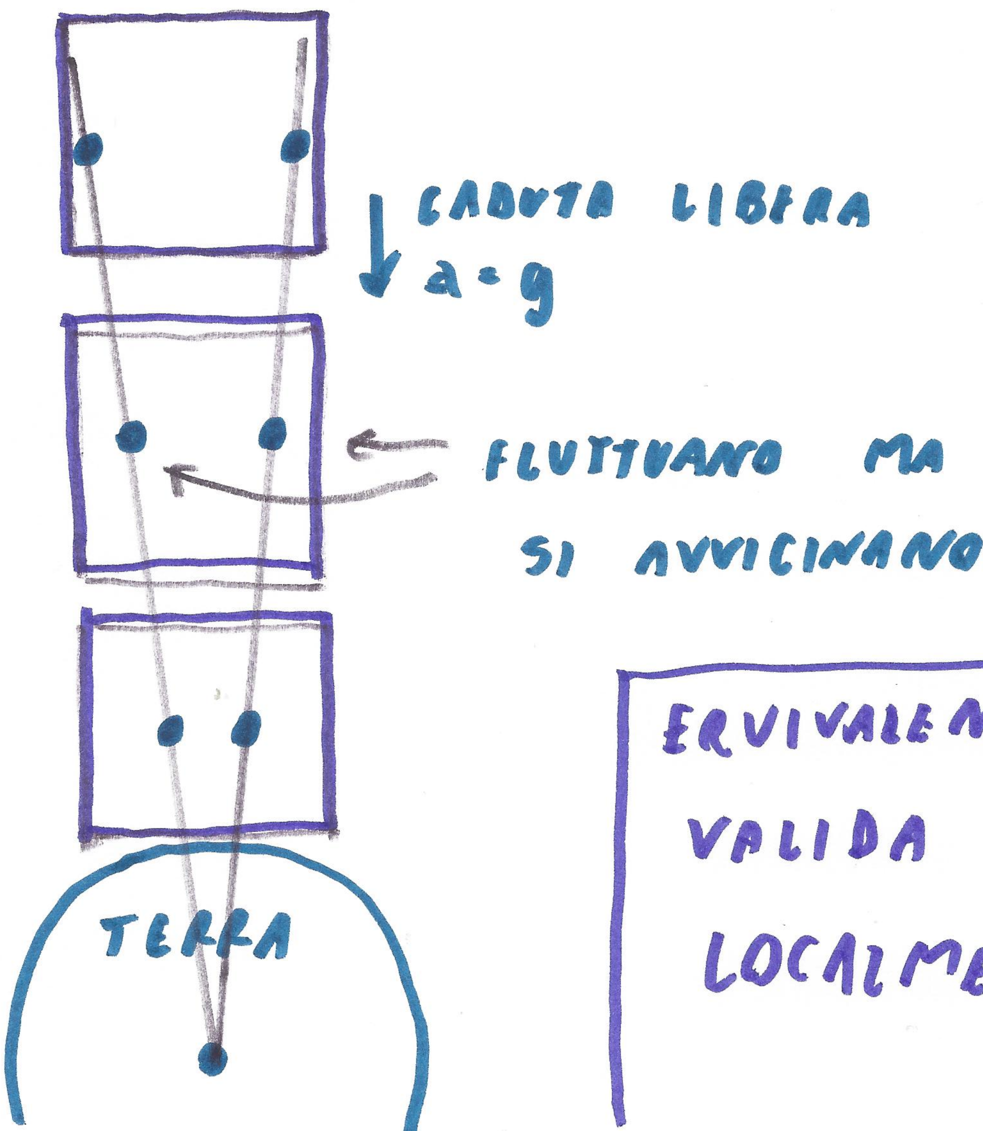
($g_{ISS} \simeq 90\% \, g$)



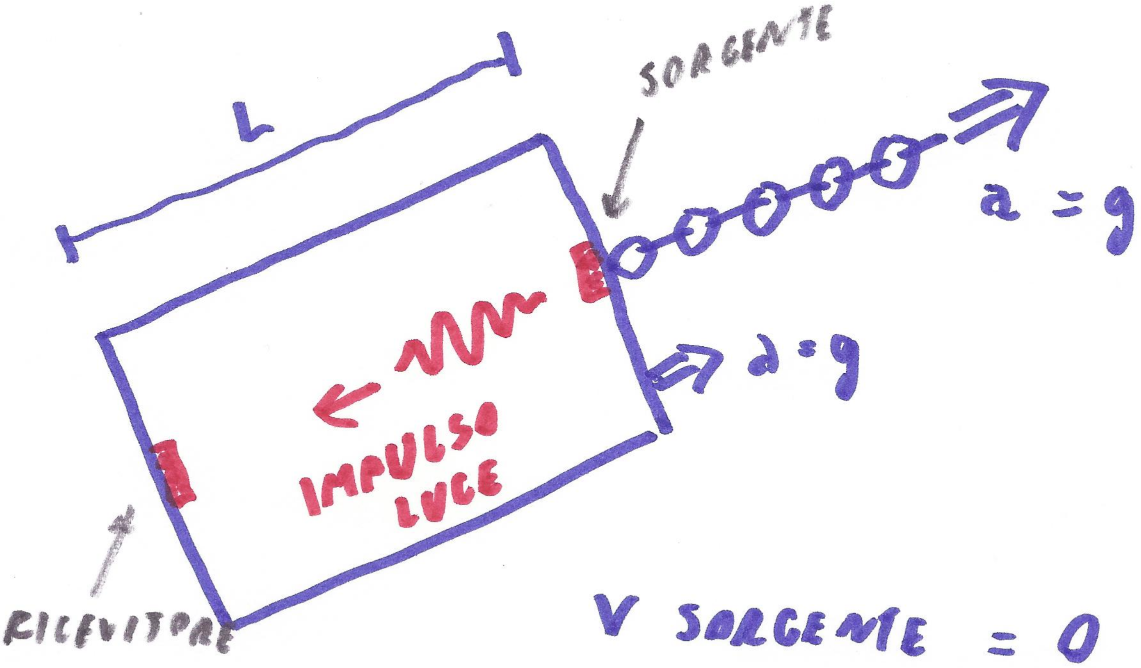
SULLA TERRA EFFETTO GRAVITÀ SOLE
~~[illegible]~~ O ALTRI CORPI CELESTI
≈ 0 (QUASI ZERO)



MAREA DOVUTA A <u>DIFFERENZE</u> G IN LUOGHI DIVERSI <u>LONTANI</u>



EQUIVALENZA VALIDA SOLO LOCALMENTE



$$V_{SORGENTE} = 0$$

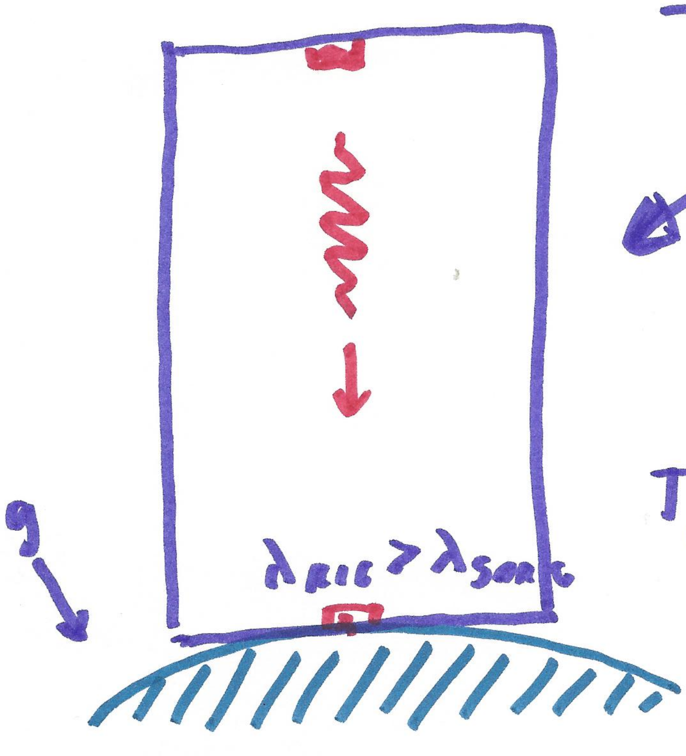
$$V_{RICEVITORE} = a\,\Delta t$$

$$\Delta t = \frac{L}{c} \neq 0$$

EFFETTO DOPPLER : $\lambda_{RICEV} > \lambda_{SORG}$

<u>EQUIVALENZA</u> :

$\boxed{\text{IDEM}}$



UNICA POSSIBILE SPIEGAZIONE

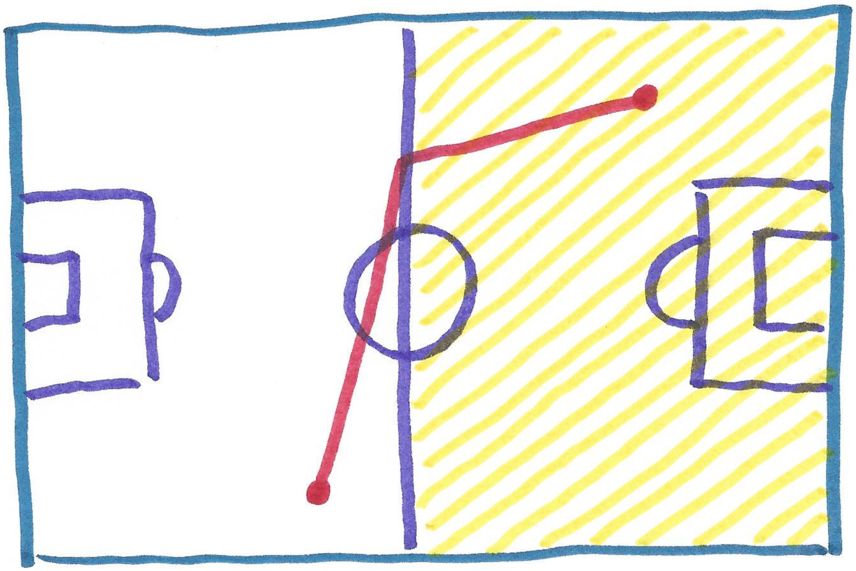
$T_{RICEVITORE}$ SCORRE PIÙ LENTAMENTE DI $T_{SORGENTE}$

PRINCIPIO FERMAT E

GENERALIZZAZIONE MECCANICA : INERZIA =

= MOTO LUNGO GEODETICHE SPAZIO-TEMPO

= MINIMO Δt PER $\vec{x}_1$ E $\vec{x}_2$ FISSATI



TRAIETTORIA

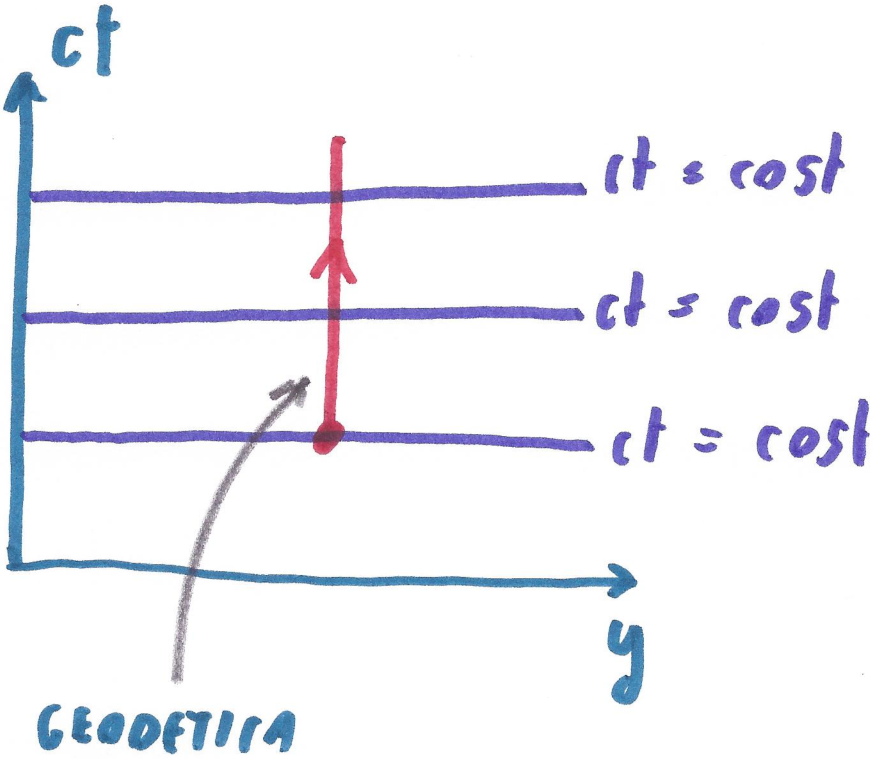
\+ VELOCE

PIENO D'ACQUA

(SI CORRE MALE)

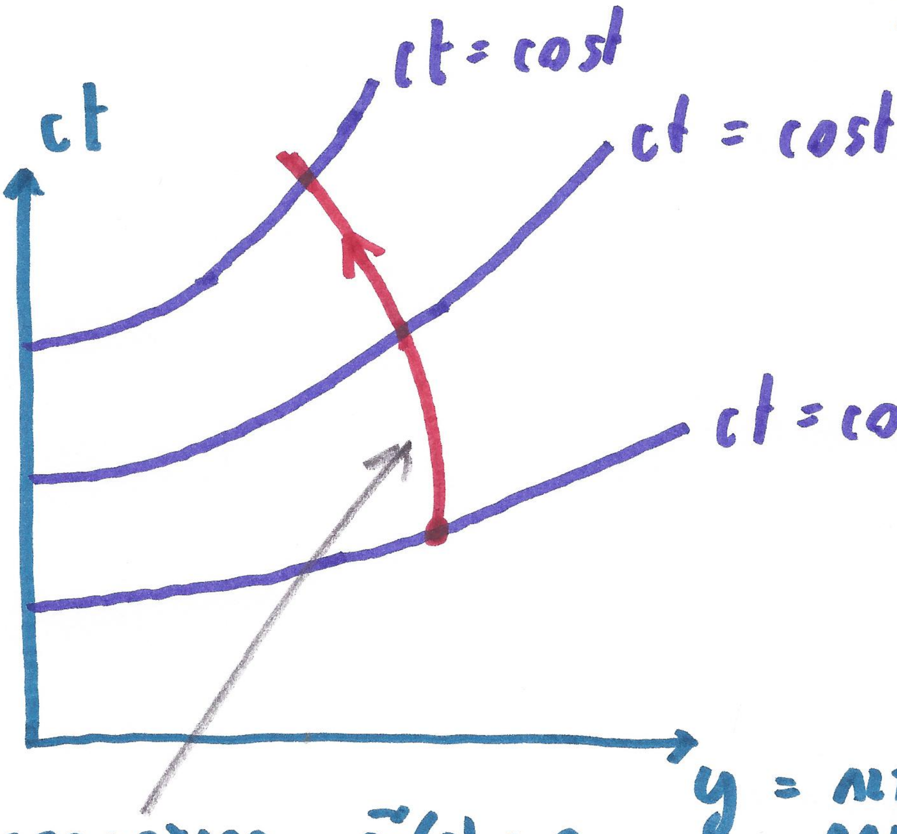
OPPURE t SCORRE

\+ VELOCE



~~vgrav~~ $\vec{v}(0) = 0 \longrightarrow \vec{v}$ RESTA 0

$y = y_0$ cost



y = ALTEZZA DALLA TERRA

GEODETICA $\vec{v}(0) = 0$

$\downarrow$

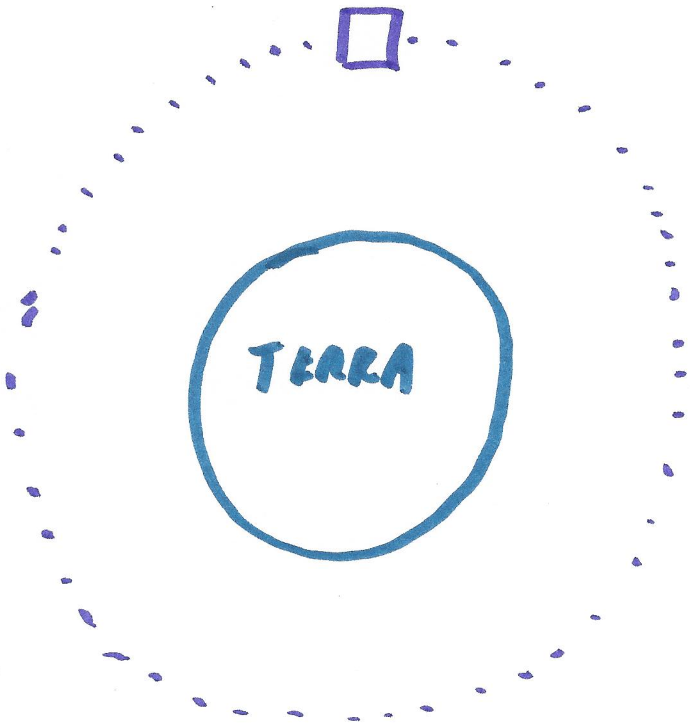
$\vec{v} \sim \vec{g}\, t$

$y \sim y_0 - \frac{1}{2} g t^2$

MECCANICA <u>NEWTONIANA</u> (NON REL.)

= CURVATURA t h

ISS = ASCENSORE CADUTA LIBERA



SU <u>INTERA ORBITA</u>

ORBITA <u>VISTA DA ISS</u>:

CRF = $2\pi R$

$v^2 = \frac{GM}{R}$ (NEWTON)

<u>DA TERRA</u> ISS È VISTA <u>CONTRATTA</u>

LUNGHEZZA ORBITA = $2\pi R \sqrt{1 - \frac{v^2}{c^2}} =$

$= 2\pi R \sqrt{1 - \frac{GM}{c^2 R}}$ $\Rightarrow$ CURVATURA <u>SPAZIALE</u> ALLA GAUSS

CRF $\neq 2\pi R$

RAGGIO LIMITE

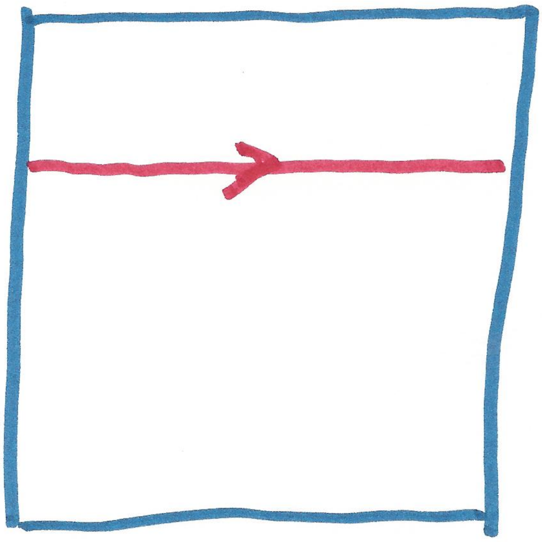
ATTORNO BUCO NERO: $1 - \frac{GM}{c^2 R} = 0$

ORBITA CONTRATTA A PUNTO ($\Rightarrow v = c$)

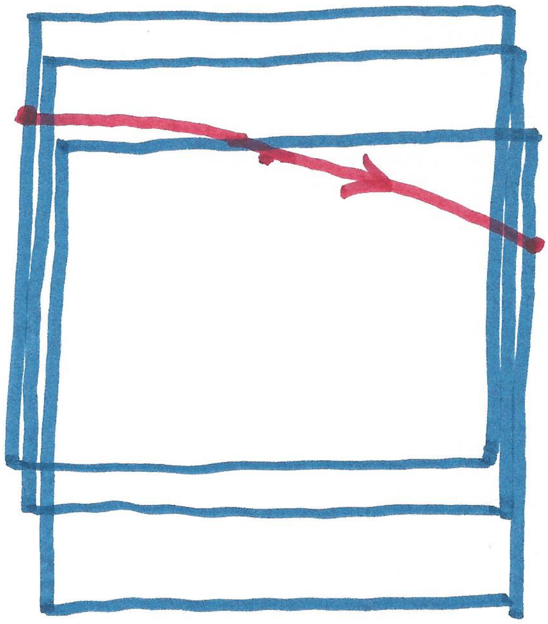
CORRETTI SOLO ORDINI GRANDEZZA !!

RAGGIO LUCE ORIZZONTALE IN ASCENSORE CADUTA LIBERA

DENTRO



VISTO FUORI



g PIEGA LA LUCE

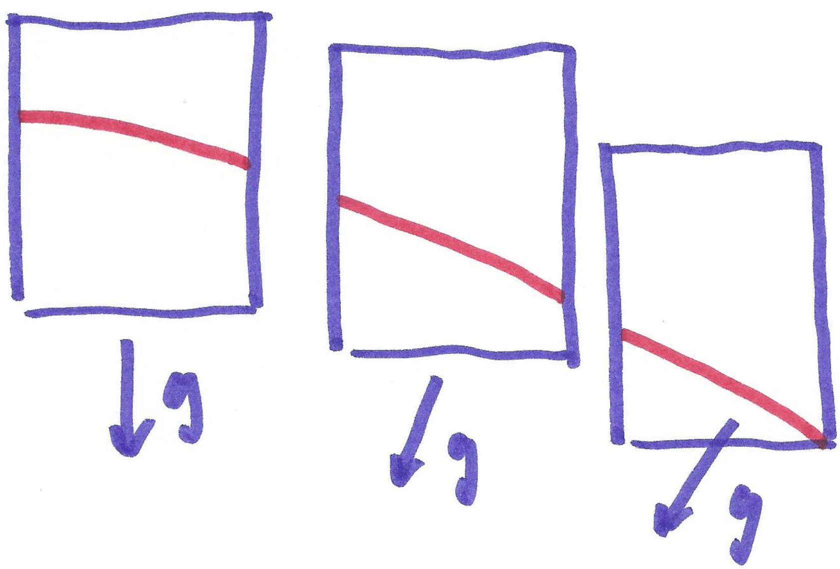
QUESTO METODO **SOTTOSTIMA** EFFETTO MISURATO (FATTORE 2

CONTA CURVATURA y^t MA TRASCURA CURVATURA SPAZIO

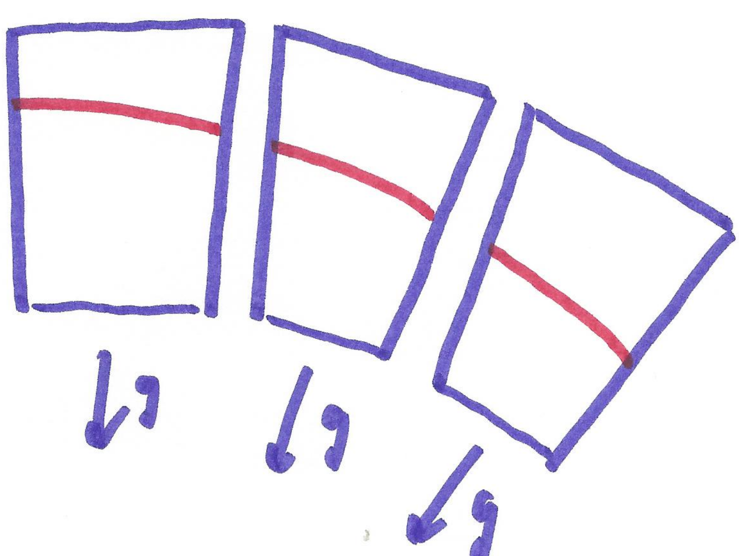
DEFLESSIONE LUCE ATTORNO SOLE

= EFFETTO <u>NON</u> LOCALE

OCCORRE "CUCIRE" EFFETTO MOLTI ASCENSORI



NON CONTO CURVATURA



CONTO CURVATURA.
ANGOLO FINALE MAGGIORE

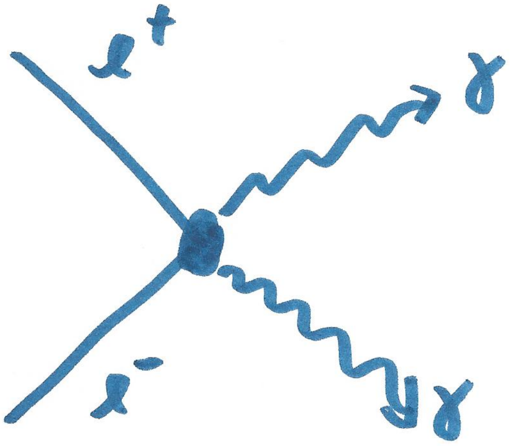
LUCE HA MASSA = 0

VIENE DEFLESSA L'ENERGIA, NON LA MASSA

$$E = Mc^2\gamma = Mc^2 + \frac{1}{2}Mv^2 + \dots$$

$$\gamma = (1-v^2/c^2)^{-1/2}$$

QUESTI IN g CONTANO



M ELETTRONI SPARISCE.

NON PUÒ "SPARIRE" LA LORO g

$M_{Fe_{53}} < 53\, M_H$ CAUSA ENERGIA LEGAME

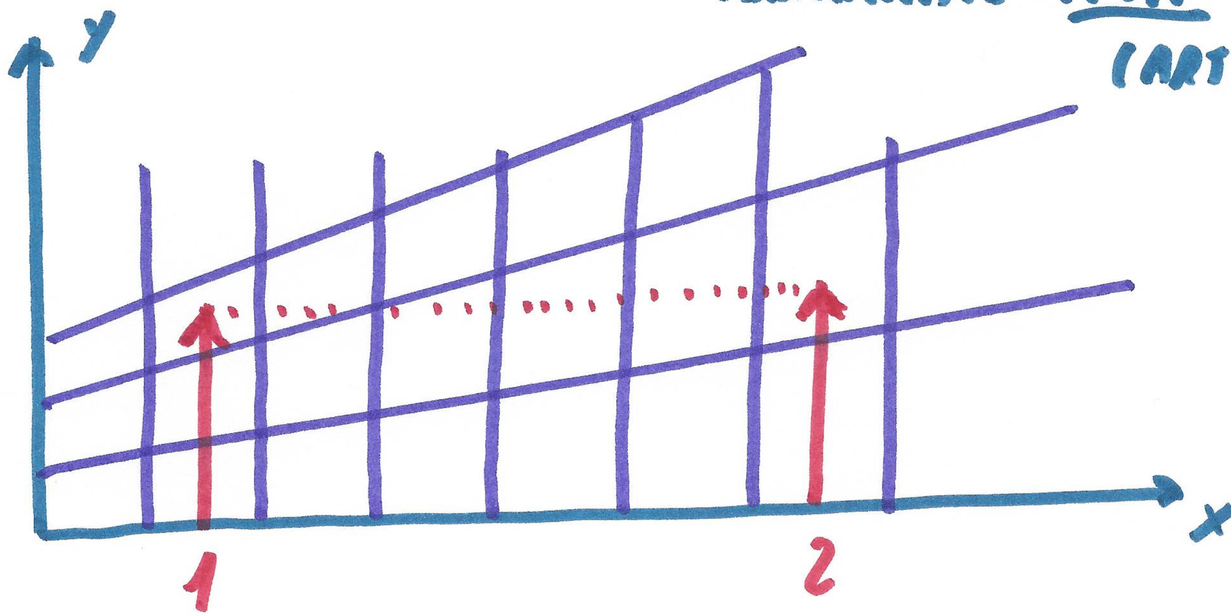
QUESTO SULLA BILANCIA SI NOTA

AZIONE - REAZIONE

$\begin{cases} g \text{ MI } \cancel{\text{ATTRAE}} \text{ ATTRAE} \rightarrow \text{EQ. GEODETICA} \\ \text{IO CREO } g \rightarrow \text{EQ EINSTEIN} \end{cases}$

TRASPORTO PARALLELO VETTORI :

COORDINATE <u>NON</u> CARTESIANE

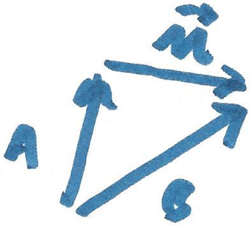


$\vec{v}(1) = (0, 2.5)$

$\vec{v}(1)_{\text{TRASPORTATO IN 2}} = (0, 1.3)$

È LO <u>STESSO</u> VETTORE, MA HA CAMBIATO COMPONENTI NELLO <u>STESSO</u>

$$T_{A\to B}(V) \equiv V'^{\alpha}$$



$$V'^{\alpha} \equiv \sum_{\beta\gamma} \Gamma^{\alpha}_{\beta\gamma} V_{\beta}\, m_{\gamma}$$

m_γ ← $\vec{B} - \vec{A}$

$\Gamma^{\alpha}_{\beta\gamma}$ ← CONNESSIONE AFFINE O SIMBOLI DI CHRISTOFFEL || MATRICE A 3 INDICI <u>NON</u> TENSORE

$$\Gamma \sim \frac{\partial g^{\alpha\beta}}{\partial x^{\gamma}} = 0 \quad \text{SE } g^{\alpha\beta} \text{ cost}$$

(SISTEMA ~~[illegible]~~ INERZIALE)

$$\Gamma \sim \frac{\partial^2 x'_{\alpha}}{} \quad \text{SE } x_B = \text{SIST. INERZIALE}$$

DERIVATA COVARIANTE

$$d\vec{V}_{(\text{NORMALE})} = \lim \frac{\vec{V}(2) - \vec{V}(1)}{2-1}$$

$$D\vec{V}_{(\text{COV})} = \lim \frac{\vec{V}(2) - TP_{1\to 2}[\vec{V}(1)]}{2-1}$$

PER CONFRONTARE $\vec{V}(2)$ CON $\vec{V}(1)$, PRIMA TRASPORTO $\vec{V}(1)$ IN 2.

PRINCIPIO INERZIA:

COVARIANTE RS

$$\frac{dv^\mu}{d\tau} = 0$$

COVARIANTE RG

$$\frac{Dv^\mu}{D\tau} = 0$$

$$\Downarrow$$

$$\frac{dv^\mu}{d\tau} + \sum_{\alpha\beta} \Gamma^\mu_{\alpha\beta} v^\alpha v^\beta = 0$$

GRAVITÀ $\sim \dfrac{\partial^2 g^{\alpha\beta}}{\partial x^\mu}$

(23)

$$v^\mu = \gamma(c, \vec{v})$$

NONRELATIVISTICO : $v^0 \gg |\vec{v}|$

$$\sum_{\alpha\beta} \Gamma^\mu_{\alpha\beta} v^\alpha v^\beta \simeq \underbrace{\Gamma^\mu_{00}\, c^2}_{\text{FORZA GRAVITÀ NEWTONIANA}}$$

$$\frac{d\vec{v}}{d\tau} \simeq \frac{d\vec{v}}{dt} = c^2 \vec{\Gamma}^{i}_{00}$$

$$\Gamma^\mu_{00} \sim \frac{\partial^0 g_{00}}{\partial x^\mu} \rightarrow g_{00} \cong 1 + 2\frac{V_{NEWT}}{c^2}$$

$$g^{\mu\nu} = \begin{pmatrix} 1 + 2\frac{V_N}{c^2} & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

SPAZIO PIATTO : SO CHE COSA VUOL DIRE "TRASPORTO PARALLELO" ≡ IDENTITÀ FISICA

SPAZIO CURVO ?

PRINCIPIO EQUIVALENZA :

ATTORNO AD OGNI PUNTO $(t, \vec{x})$ ESISTE UNA ZONA IN CUI UNA TRASF. DI COORD. INDIVIDUA UN SISTEMA INERZIALE

IN QUEL SISTEMA IL TRASPORTO PARALLELO È L'IDENTITÀ

DOPO UN TRASPORTO PARALLELO INFINITESIMO , OCCORRE IDENTIFICARE UN NUOVO SISTEMA LOCALMENTE INERZIALE , E CONTINUARE .

SPAZIOTEMPO PIATTO :

S. INERZIALE GLOBALE

$\Gamma^{\alpha}_{\beta\gamma} = 0$ OVUNQUE

$\hookrightarrow \dfrac{\partial g^{\alpha\beta}}{\partial x} = 0 \qquad g^{\alpha\beta} = \text{cost}$

S. INERZIALE LOCALE

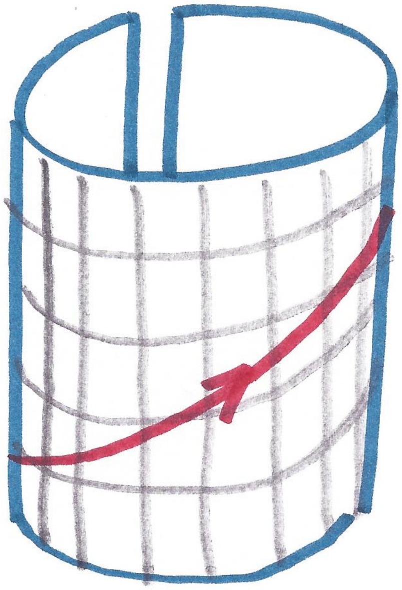
IN <u>UN</u> PUNTO

$$\begin{cases} \Gamma^{\alpha}_{\beta\gamma} = 0 \longrightarrow \dfrac{\partial g^{\alpha\beta}}{\partial x^{\gamma}} = 0 \\ \dfrac{\partial \Gamma^{\alpha}_{\beta\gamma}}{\partial x} \neq 0 \longrightarrow g^{\alpha\beta} = \end{cases}$$

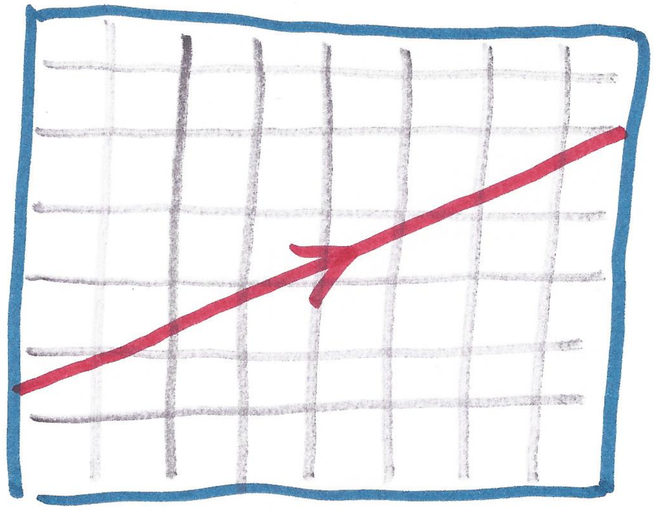
$= \text{cost} + O(x^2)$

CURVATURA

CURVATURA "CARTOGRAFICA" STILE GAUSS-RIEMANN

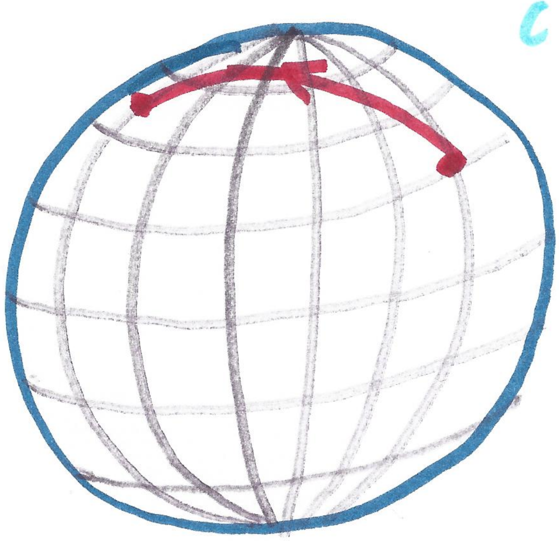


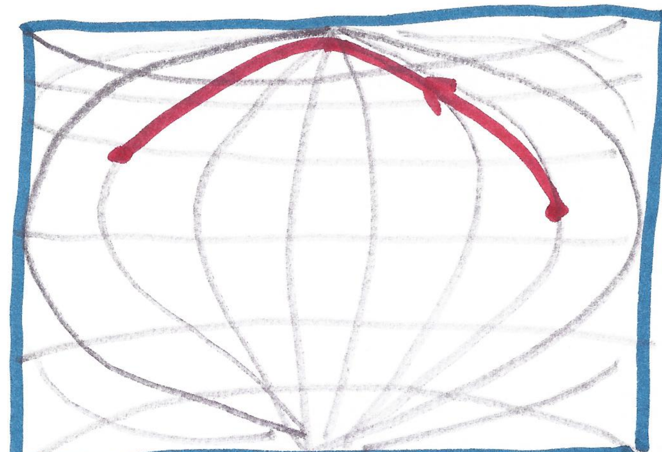
NON CURVO



= GEODETICA INERZIALE

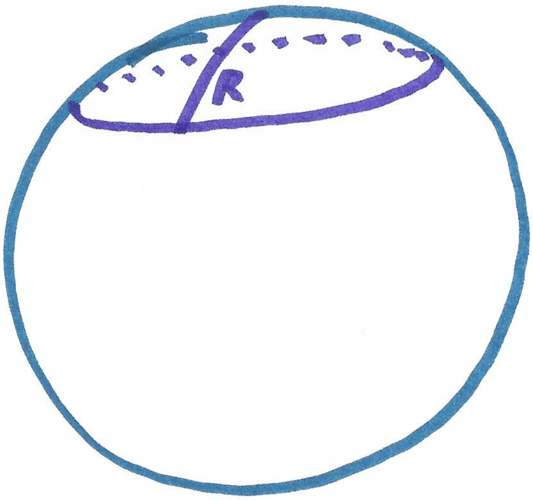
CURVO





GEODETICA REALE NON APPARE TALE SUL PLANISFERO

CRITERIO GAUSS CURVATURA INTRINSECA



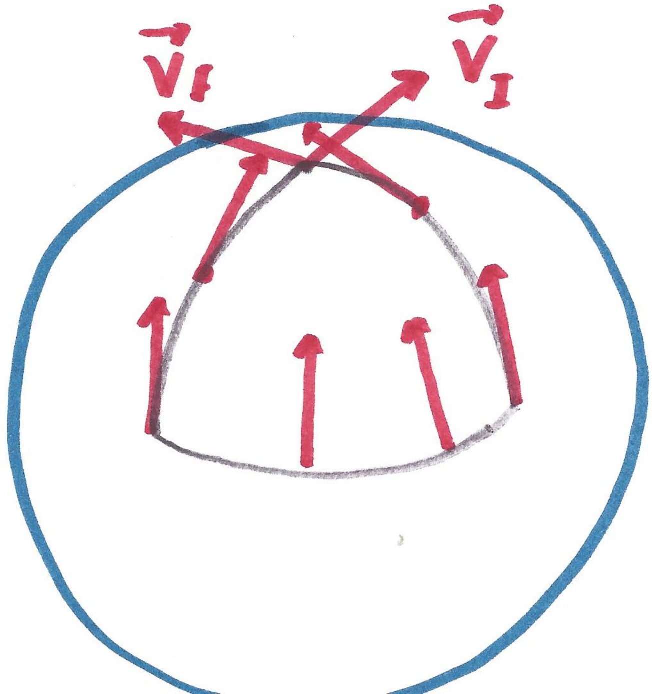
$CRF < 2\pi R$

PER $R \to 0$

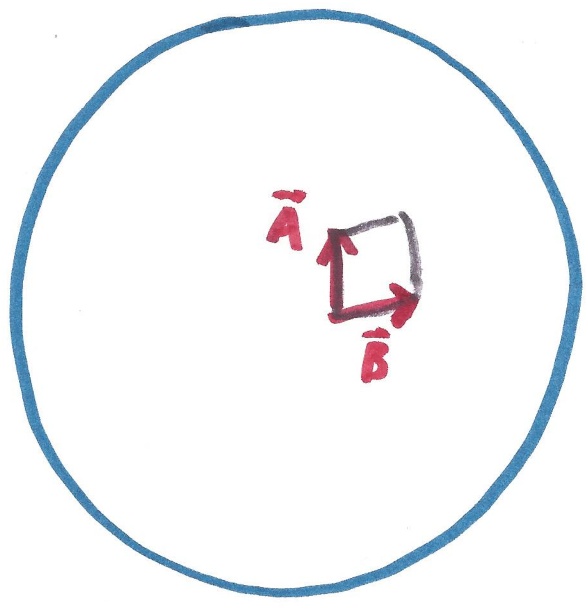
DA $|CRF - 2\pi R|$ SI RICAVA

$R_{CURVATURA} (= R_{SFERA})$

STESSO RISULTATO DA CRITERIO RIEMANN



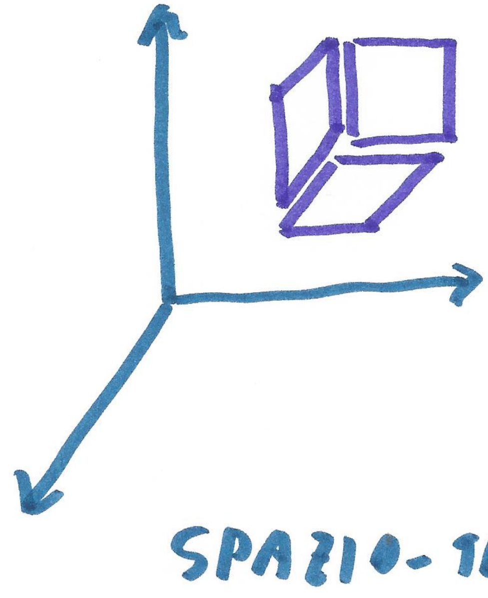
$\vec{V}_F \neq \vec{V}_I$



TRASPORTO PARALLELO DI $\vec{V}$ LUNGO UN RETTANGOLO <u>PICCOLO</u> INDIVIDUATO DA $\vec{A}, \vec{B}$

$$\Delta V^\alpha \equiv \sum_{\beta\gamma\delta} R^\alpha{}_{\beta\gamma\delta} V^\beta A^\gamma B^\delta$$

$\hookrightarrow$ DA $R^\alpha_{\beta\gamma\delta}$ SI ESTRAE R_{CURV} (= R_{SFERA})



SPAZIO 3-DIM :

3 POSSIBILI ORIENTAZIONI RETTANGOLI

SPAZIO-TEMPO 4-DIM : 6 ORIENTAZ.

tx, ty, tz, xy, xz, yz

\+ SEMPLICI :

TENSORE RICCI :

$$R_{\alpha\beta} = \sum_{\gamma\delta} R_{\alpha\gamma\delta\beta}$$

TENSORE EINSTEIN $G_{\alpha\beta}$:

$$R_{\alpha\beta} - \text{Traccia } R_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} \sum_{\gamma} R$$

↳ MOLTO SIMILE A $R_{\alpha\beta}$ MA

Traccia $G_{\alpha\beta} = 0$

$R_{\alpha\beta}$ ~ "MEDIA" SUI RETTANGOLI $\vec{A}\,\vec{B}$ $\perp \vec{B}$ E SUI VETTORI $\vec{V}$

$$\left.\begin{matrix} R_{\alpha\beta\gamma\delta}\\ R_{\alpha\beta}\\ G_{\alpha\beta}\end{matrix}\right\} \begin{aligned} &\sim \frac{\partial^2 g^{\alpha\beta}}{\partial x^\gamma \partial x^\delta} \\ &\sim \frac{\partial \Gamma^\alpha_{\beta\gamma}}{\partial x^\delta} + \sum_\mu \Gamma^\alpha_{\beta\mu}\Gamma^\mu_{\gamma\delta}\end{aligned}$$

CURVATURA

NON LINEARITÀ

IDENTITÀ BIANCHI

$$\sum \frac{D\, R_{\alpha\beta\gamma\delta}}{D\, x_{\mu}} = 0 \qquad \nwarrow \text{ TIPO } \nabla \cdot (\vec{\nabla} \wedge \vec{A}) = 0$$

$$\Downarrow$$

$$\sum_{\alpha} \frac{D\, R_{\alpha\beta}}{D\, x_{\alpha}} = 0$$

$\nwarrow$ TIPO

$$\nabla \cdot \vec{R}_{\beta} = 0$$

CON $\vec{R}_{\beta} \equiv R_{i\beta}$

E IDEM PER $G_{\mu\nu}$

FONDAMENTALE PERCHÉ

$$\sum_{\alpha} \frac{D\, T_{\alpha\beta}}{D\, x_{\alpha}} = 0 \qquad \longleftarrow \text{ TENSORE ENERGIA-IMPULSO}$$

$$\begin{pmatrix} \frac{dE}{dV} & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \longleftarrow \begin{pmatrix} \frac{dE}{dV} & \phi_x(E) & \phi_y(E) & \phi_z(E) \\ & \phi_x(p_x) & \phi_y(p_x) & \phi_z(p_x) \\ \vdots & & \vdots & \\ \vdots & & \vdots & \end{pmatrix}$$

(MATERIA STAZIONARIA)

1^a IPOTESI EINSTEIN

$$G_{\mu\nu} = -\text{cost}\ T_{\mu\nu}$$

SI DETERMINA COL CASO NEWTON NR

POI EINSTEIN AGGIUNGE

$$G_{\mu\nu} = -\text{cost}\ T_{\mu\nu} + \underbrace{\lambda g_{\mu\nu}}$$

costante cosmologic

GRAVITÀ REPULSIVA DEL VUOTO (IMPEDISCE AUTOCOLLASSO)

SCOPERTO UNIVERSO IN ESPANSIONE (HUBBLE

EINSTEIN TOGLIE $\lambda g_{\mu\nu}$

("LA PIÙ GRANDE SCIOCCHEZZA DELLA STORIA DELLA FISICA")

MOLTI ANNI DOPO (~ ANNO 2000) SCOPERTO CHE

$v_{ESPANSIONE}$ AUMENTA $\longrightarrow$ TORNA $\lambda g_{\mu\nu}$

PIANETA ISOLATO

$$T_{\mu\nu} = 0$$

$$\begin{matrix} T_{\mu\nu} \\ \neq 0 \end{matrix}$$

$\downarrow$

$$G_{\mu\nu} = 0$$

MA

$$R_{\alpha\beta\mu\nu} \neq 0$$ (LA GRAVITÀ C'È)

$$\hookrightarrow \frac{\partial \Gamma^{\alpha}_{\beta\gamma}}{\partial x_{\delta}} \cancel{\neq} \neq 0$$

RICORDA: LIM NR: $\Gamma^{\alpha}_{00} \sim F^{\alpha}_{NEWTON}$

$G_{\mu\nu} = 0$ SIGNIFICA

$$\sum \frac{\partial \Gamma}{\partial x} = 0$$

SITUAZIONE TIPO TEOREMA DI GAUSS

$$\nabla \cdot \vec{E} = \rho_{ELETTRICA} = \frac{dQ}{}$$

NEI VUOTO

$$\sum_\alpha \frac{\partial E_\alpha}{\partial x_\alpha} = Q \qquad E \sim 1/R^2$$

DALLA CARICA ESCE FLUSSO

$$\phi(E) = Q_{TOT}$$

EQ. EINSTEIN DICONO NEL VUOTO

$$\sum_\alpha \frac{\partial \Gamma^\alpha_{00}}{\partial x^\alpha} = 0 \qquad \left(\begin{array}{c} \text{LIM NR} \\ \text{CONTA SOLO } \Gamma^\alpha_{00} \end{array} \right)$$

CONSERVAZIONE FLUSSO CAMPO NEWTONIANO

$$\vec{F}_{NEWTON} \sim 1/R^2$$

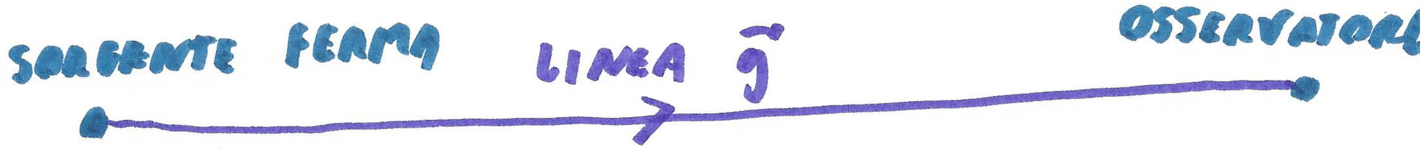
E CON MATERIA

$$\sum_\alpha \frac{\partial \Gamma^\alpha_{00}}{\partial x^\alpha} = -\text{cost} \frac{dE}{dV} \simeq -\text{cost} \frac{dM}{dV}$$

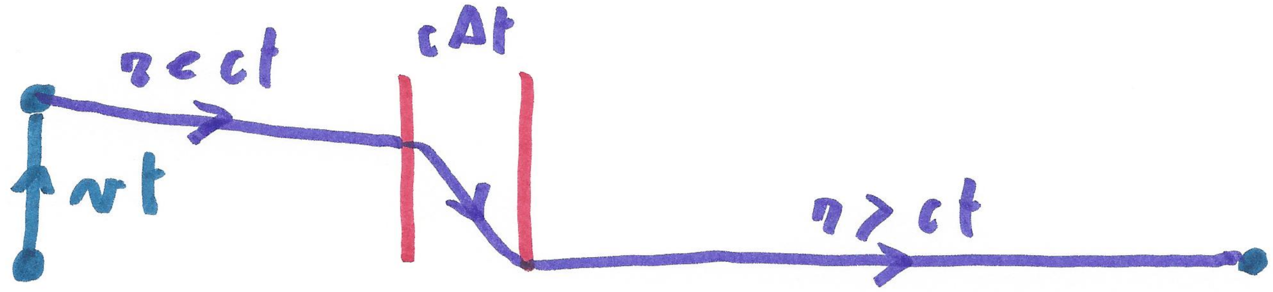
COME PER $\vec{E}_{GAUSS}$

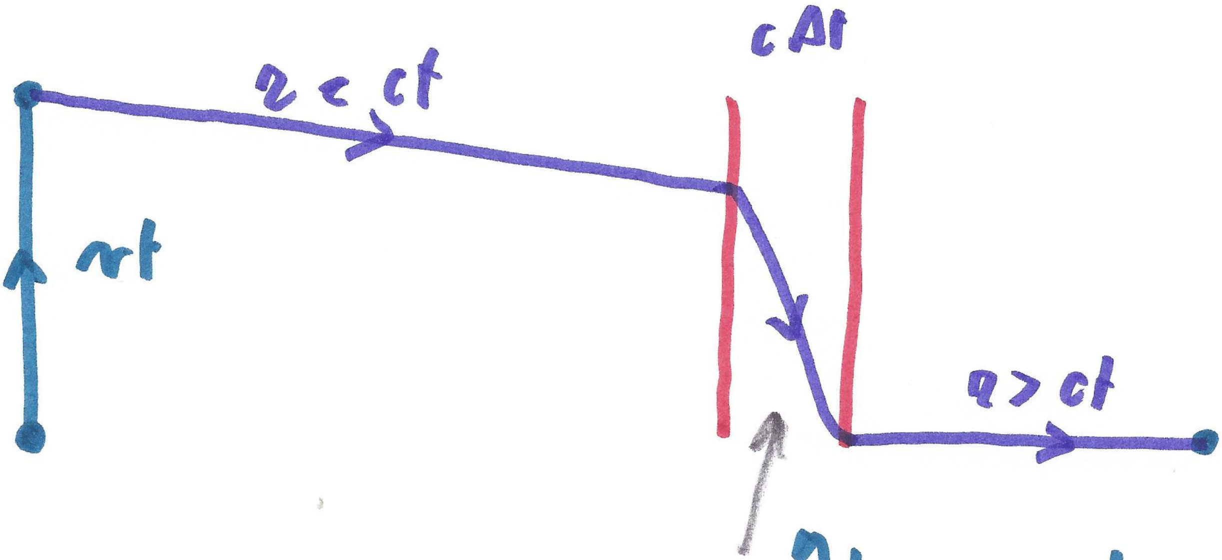
CONSERVAZ. FLUSSO NEL VUOTO +

+ C FINITA = ONDE G



ACCELERA IN UN TEMPO BREVE Δt

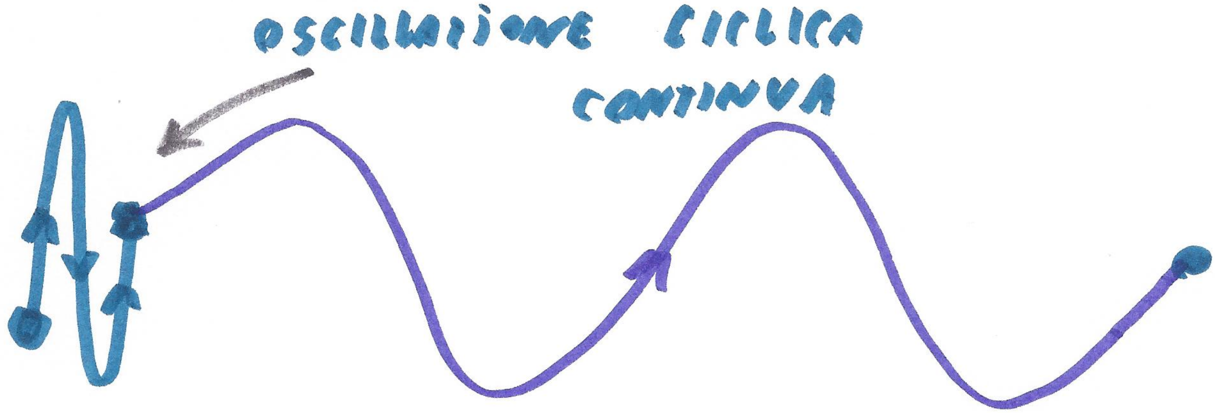


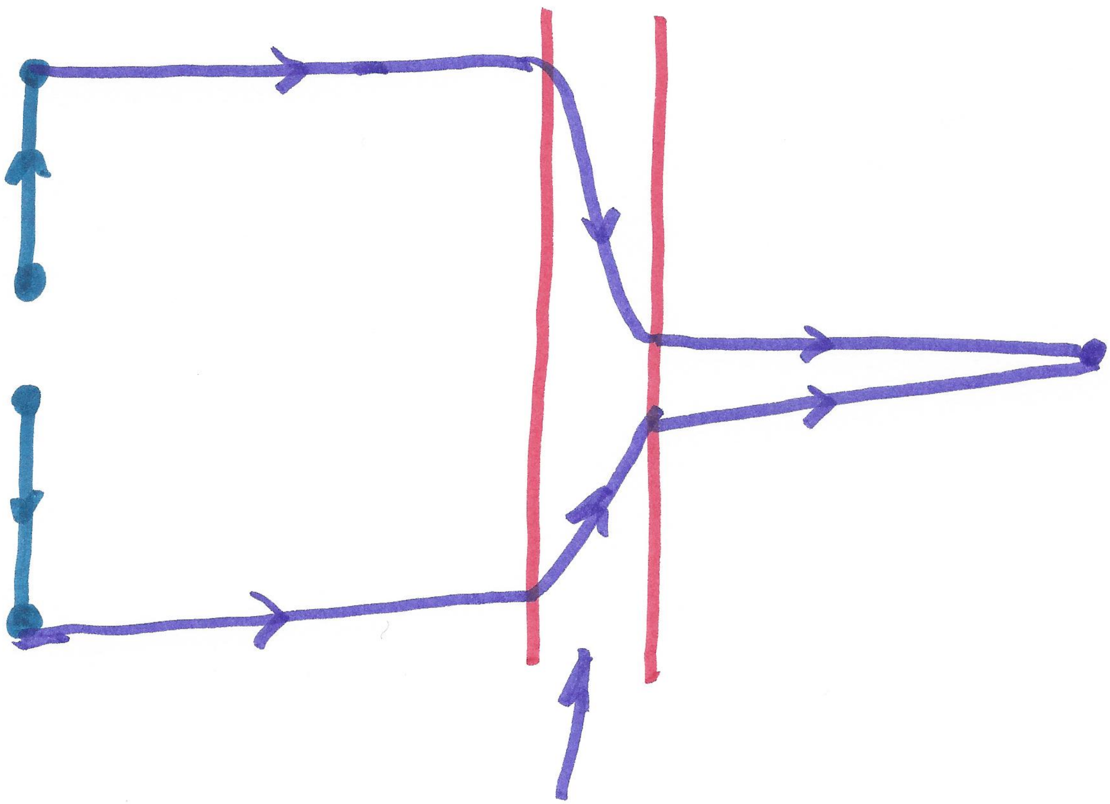


$$\frac{g_\perp}{g_\parallel} \simeq \frac{vt}{c\Delta t} \propto r$$

$g_\parallel \sim 1/r^2$

$g_\perp \sim 1/r$





MOTI SORGENTE ED ONDE
QUADRUPOLARI
(CONTRAZIONI - DILATAZIONI,
SENZA POLARIZZAZIONE)